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DISTRIBUTED INJECTION OF A GAS INTO A HYPERSONIC FLOW

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UDC 532.526

Distributed surface injection of a gas is used to reduce heat flows to the surface of aircraft traveling at high supersonic velocities. The injection changes the effective form of the surface and can therefore be used to create aerodynamic forces and moments. The latter case is characterized by velocities normal to the injection surface which are an order of magnitude greater than the vertical velocity in the boundary layer on an impermeable surface. Flow regimes with intensive injection have been studied in several investigations, a survey of which is offered in [1]. At the same time, for the goal of protection from heating, it is optimum if the flow rate of the injected gas is comparable to the flow rate in the boundary layer on an impermeable surface, since the intensity of the injection ensures a reduction in heat flux in the dominant term. In this case, flow near the permeable surface is described by a system of boundary-layer equations. Hypersonic flows are characterized by the highest heat fluxes, and this is particularly true for the regime of strong hypersonic interaction.

Studies of flows for this regime have been limited mainly to examining problems with boundary conditions, which provide for a reduction in the system of boundary-layer equations to a system of ordinary differential equations [2]. At the same time, the distribution of injection rate realized in practice makes it necessary to solve problems which are not self-similar. An example of the solution of such problems is given in the present study.

There is yet one more circumstance which makes the study of flows with injection particularly important. In classical boundary-layer theory, there are two types of singularities in the solution. These singularities are connected with the vanishing of skin friction and with alteration of the structure of the flow. In the first case, friction decreases to zero and a region of reverse currents is formed (the boundary layer separates) due to an unfavorable pressure gradient. In the second case, distributed injection causes friction to vanish and a region of inviscid boundary flow to form (the boundary layer is detached). The structure of flow in the boundary layer is determined by diffusion and convection associated with vorticity. At large Reynolds numbers, the distance over which the vorticity diffuses from the solid surface is much less than the distance over which the vorticity is transported along the surface by convection [3]. Stagnation of the fluid under the influence of an unfavorable pressure gradient leads to development of the convective mechanism of vorticity transport from the surface and to restructuring of the flow in the boundary layer. Such convection also develops as a result of surface injection. The solutions of the system of boundary-layer equations near points of zero friction were described mathematically in [4, 5]. Analysis of these solutions showed that a large unfavorable pressure gradient, induced by the displacement thickness in the external flow, develops in the vicinity of points of zero skin friction. By allowing for the interaction of the boundary-layer flow with the external flow, it was possible for investigators to obtain a smooth solution which passed through the separation point in supersonic [6, 7] and subsonic [8] flows. It later turned out that allowing for an induced pressure gradient in a composite system of boundary-layer equations makes it possible to also eliminate the singularity for the solution which describes flow with distributed injection [9]. The solution obtained in [9] corresponded to the regime of weak interaction, and the induced pressure gradient began to have an appreciable effect only after skin friction was reduced to nearly zero. The strong interaction regime is characterized by the fact that the boundary-layer flow and the inviscid external flow influence each other along the entire surface of the body. Thus, if it exists at all, the phenomenon of boundary-layer detachment should have several features which will distinguish it from the analogous phenomenon in the weak interaction regime. It is the analysis of these features which is the focus of this article.

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1. We are examining the hypersonic flow of a viscous, heat-conducting gas about a plane surface. The origin of the Cartesian coordinate system coincides with the leading edge. The axis $x\ell$ is directed along the surface, while the axis $y\ell$ is directed normal to the surface (Fig. 1). In accordance with [10], the regime of strong interaction is realized at

$$M_\infty \to \infty,\ M_\infty \tau \to \infty,$$

where M_∞ is the Mach number in the undisturbed flow; τ is the thickness of the boundary layer δ^* referred to the length of the surface ℓ. In conformity with the usual estimates of the theory of strong interaction in a boundary layer (region 2 in Fig. 1), we introduce the following notation for the components of the velocity vector, density, pressure, total enthalpy, and absolute viscosity: $u_\infty u_1$, $\tau u_\infty v_1$, $\tau^2\rho_\infty\rho_1$, $\tau^2\rho_\infty u_\infty^2 p_1$, $(u_\infty^2/2)g_1$, $\mu_0\mu_1$. Here, the subscript ∞ denotes parameters of the undisturbed incoming flow, while the subscript 0 pertains to the viscosity coefficient corresponding to the stagnation temperature. The parameter τ, characterizing the thickness of the boundary layer, is expressed through the Reynolds number $\tau = \mathrm{Re}_0^{-1/4}$, $\mathrm{Re}_0 = \rho_\infty u_\infty \ell/\mu_0$. It is assumed that the surface of the plate is permeable and that the gas injected along a normal to the plate is of the same composition as the gas in the incoming flow. The velocity of the gas is equal to $\tau u_\infty v_w$.

The system of equations of the boundary layer, written in Dorodnitsyn variables, has the form

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1^*}{\partial \eta} = 0,\quad \frac{\partial p_1}{\partial \eta} = 0,\quad v_1^* = \rho_1 v_1 + u_1\frac{\partial \eta}{\partial x},$$
$$u_1\frac{\partial u_1}{\partial x} + v_1^*\frac{\partial u_1}{\partial \eta} + \frac{1}{\rho_1}\frac{\partial p_1}{\partial x} = \frac{\partial}{\partial \eta}\left(\rho_1\mu_1\frac{\partial u_1}{\partial \eta}\right),$$
$$u_1\frac{\partial g_1}{\partial x} + v_1^*\frac{\partial g_1}{\partial \eta} = \frac{\partial}{\partial \eta}\left\{\rho_1\mu_1\left[\frac{1}{\sigma}\frac{\partial g_1}{\partial \eta} + \left(1-\frac{1}{\sigma}\right)\frac{\partial}{\partial \eta}(u_1^2)\right]\right\},\tag{1.1}$$
$$p_1 = \frac{(\gamma-1)}{2\gamma}(g_1-u_1^2)\rho_1,\quad \eta = \int_0^y \rho_1\,dy,\quad \delta_1 = \rho_1^{-1}d\eta,$$
$$\rho_1\mu_1 = \frac{2\gamma}{(\gamma-1)}p_1(g_1-u_1^2)^{n-1},\quad u_1(x,0) = 0,$$
$$g_1(x,0) = g_w,\ v_1^*(x,0) = v_w\rho_{1w},\ u_1(x,\infty) = 1,\ g_1(x,\infty) = 1$$

(σ is the Prandtl number).

In accordance with the theory of strong interaction [10], the pressure distribution $p_1(x)$ which goes into the boundary-value problem depends on the displacement thickness of the boundary layer δ_1. To determine this dependence, it is necessary to study the inviscid flow in region 1 (see Fig. 1), located between the shock wave and the external boundary of the boundary layer. The flow in region 1 is described by the hypersonic theory of small perturbations [11]. We will use the following approximate expression for subsequent analysis

$$p_1 = \frac{\gamma+1}{4}\left(\frac{d\delta_1}{dx}\right)^2,\tag{1.2}$$

this expression being given by the shear-wedge method.

The substitution of variables $\xi = x^{1/4}$, $\lambda = \eta\xi^{-1}[(\gamma-1)/8\gamma p(0)]^{1/2}$, $u_1 = \partial\psi/\partial\eta$, $v_1^* = -\partial\psi/\partial x$, $\psi = \xi f[8\gamma p(0)/(\gamma-1)]^{1/2}$, $p_1 = \xi^{-2}p$, $\rho_1 = \xi^{-2}\rho$, $\delta_1 = \xi^3\delta$, $g_1 = g$ reduces boundary-value problem (1.1)-(1.2) to the form

$$(Nf'')' + ff'' + \frac{\gamma-1}{\gamma}\left(1-\frac{\xi p^\cdot}{2p}\right)(g-f'^2) = \xi(f'f'^\cdot - f^\cdot f''),$$
$$\left(\frac{N}{\sigma}g'\right)' + fg' + \left[N\left(1-\frac{1}{\sigma}\right)f'^2\right]' = \xi(f'g^\cdot - f^\cdot g'),$$
$$f'(\xi,0) = 0,\ g(\xi,0) = g_w,\ f(\xi,0) = f_w,\ f'(\xi,\infty) = 1,\ g(\xi,\infty) = 1,\tag{1.3}$$
$$\delta = 2\left(\frac{\gamma-1}{2\gamma p}\right)^{1/2}\left[\frac{p(0)}{p}\right]^{1/2}\int_0^\infty (g-f'^2)\,g\eta,\quad N = \frac{p}{p(0)},$$

$$p = \frac{\gamma+1}{2}\left[\frac{3}{4}\delta + \frac{1}{4}\xi\frac{d\delta}{d\xi}\right]^2,\ f_w = -\left(\frac{2\gamma}{\gamma-1}\right)^{1/2}\frac{2v_w}{g_w\xi[p(0)]^{1/2}}\int_0^\xi p\xi\,d\xi,\quad f' = \frac{\partial f}{\partial\lambda},\quad f^\cdot = \frac{\partial f}{\partial\xi}.$$

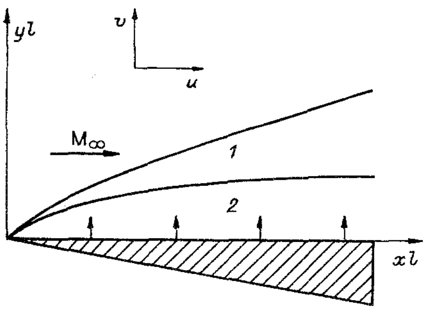

Fig. 1

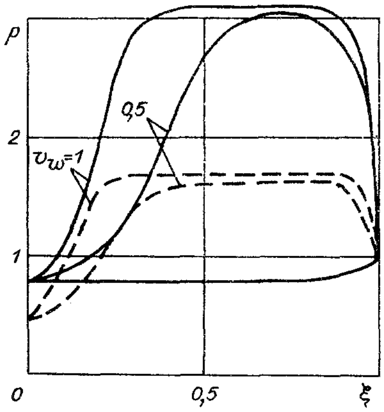

Fig. 2

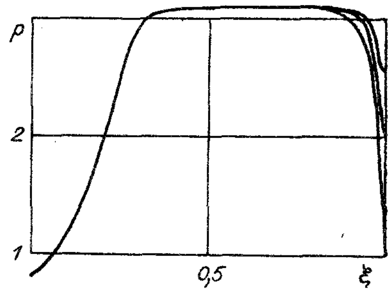

Fig. 3

The pressure distribution $p(\xi)$ in (1.3) is unknown beforehand and is found during the solution of the problem. The presence of the induced pressure gradient imparts new properties to the solution of the parabolic system of boundary-layer equations. These properties are related to the transfer of disturbances upflow or to the dependence on the boundary condition assigned downflow [12]. For example, the auxiliary boundary condition assigned on the bottom edge $p(\xi = 1) = B$ makes it possible to obtain a unique solution for boundary-value problem (1.3). Efficient difference schemes have been developed to numerically solve boundary-value problems of this type, and we will use the method in [13] here. The procedure for obtaining the solution involves assigning a certain velocity and pressure field in the region $(0 \leqslant \xi \leqslant 1;\ 0 \leqslant \lambda < \infty)$. Linearized boundary-value problem (1.3) is subsequently solved with a known pressure gradient and known distributions of pressure and displacement thickness $\delta^i(\xi)$. As a result, we find a new distribution of the displacement thickness $\delta(\xi)$ which differs from the original distribution $\delta^i(\xi)$. The next stage of calculation involves finding the correction $\Delta(\xi)$ for the displacement thickness distribution. We do this through the use of a linear second-order differential equation in which the inhomogeneous term is proportional to the difference $\delta^i(\xi) - \delta(\xi)$. The calculational procedure is repeated with a new displacement thickness distribution $\delta^{i+1} = \delta^i + \Delta$ and the corresponding distributions of pressure and pressure gradient until the difference $\delta^{i+1} - \delta^i$ becomes sufficiently small. Thus, it is also possible to calculate flow in a boundary layer with reverse currents by using approximate differences in the approximation of the convective derivatives.

2. Numerical solutions of boundary-value problem (1.3) were obtained with $\sigma = n = 1$ and $\gamma = 1.4$. Figure 2 shows results of calculations of the function $p(\xi)$ which correspond to a fixed value of the parameter $B = 1.02$ (proportional to the bottom pressure gradient) and several values of the parameter v_w, proportional to the sonic velocity. The solid curves correspond to the temperature factor $g_w = 1$, while the dashed lines correspond to $g_w = 0.5$. We should point out the qualitative difference between the solutions corresponding to flow about impermeable and permeable surfaces. The first solution ($v_w = 0$) is characterized by constancy of the function $p(\xi)$ nearly everywhere, except for the region adjacent to the bottom edge ($\xi = 1$). The second type of solution ($v_w \neq 0$) is characterized by regions of rapid increase near the leading edge and nearly constant values and changes in the vicinity of the bottom edge. In the region of nearly constant values, the function p depends only slightly on the injection rate and is determined by the temperature factor. A reduction in g_w in this region is accompanied by a reduction in the maximum of $p(\xi)$. It is understood that these conclusions pertain only to the investigated range of the bottom pressure gradient, in which the flow does not contain any regions of reverse currents.

Figure 3 shows results of study of the effect of B on the characteristics of the flow. Here, the function $p(\xi)$ was obtained with $v_w = 1$, $g_w = 1$. It should be noted that, with a change in B, the value of $p(\xi)$ changes only near the bottom edge. It was shown in [11] that the solution of boundary-value problem (1.3) written (for example) for $p(\xi)$ can at $\xi \to 0$ be written in the form of a series containing an eigenfunction of the form $C\xi^a$, where a is the eigenvalue and the constant C is determined by the condition $p(\xi = 1) = B$. The study of flows with injection given by the condition $f_w(\xi) = -F$ in [14] showed that an increase in F leads to a reduction in a. Accordingly, there is an increase in the rate of transfer of disturbances upflow; in particular, with a fixed value of bottom pressure gradient, pressure increases at any point near the leading edge. The slight dependence of the solution on the

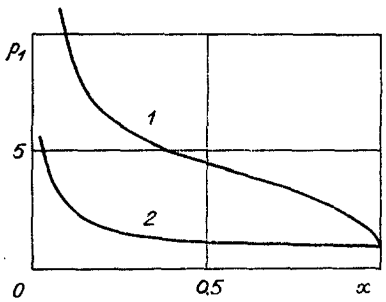

Fig. 4

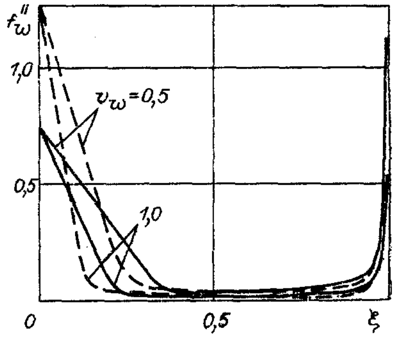

Fig. 5

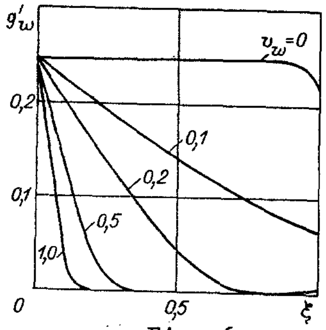

Fig. 6

bottom pressure gradient (Fig. 3) can be attributed to the fact that, at $\xi \to 0$, the injection regime being studied, v_w = const or $f_w(\xi) = O(\xi)$, is characterized by an appreciably lower injection velocity near the leading edge than the regime corresponding to the condition f_w = const.

The results of the calculations allow us to conclude that a region of reverse flows (or a change in the sign of skin friction with an increase in the parameter B) is first seen near the bottom edge, i.e., at $\xi = 1$. To explain this fact, we need to return to the definition of the function $p(\xi)$. This function is the ratio of the pressure distribution to the similarity distribution that corresponds to flow about a semi-infinite impermeable surface. The pressure distributions $p_1(x)$, expressed in similarity variables, are shown in Fig. 4 [curve 2 corresponds to the similarity solution ($v_w = 0$, $g_w = 1$), while curve 1 was obtained with the same value of the temperature factor and $v_w = 1$]. It can be seen that the pressure distributions are monotonic and correspond to a negative pressure gradient over the entire surface of the body. An increase in B leads to a change in the pressure distribution near the leading edge. Specifically, it leads to the appearance of zero or negative skin friction associated with a change in the sign of the pressure gradient at $\xi = 1$.

Comparing our results with the results of study of a flow with uniformly distributed injection for the regime of weak interaction [9], it can be noted that the strong-interaction regime is characterized by a shift in the region of pressure increase (positive pressure gradient) toward the bottom edge.

Figure 5 shows results of calculations of the function $f''_w(\xi)$. Here, as for $p(\xi)$ (see Fig. 2), we can distinguish three characteristic regions: rapid reduction in the function $f''_w(\xi)$ near the leading edge, near-constant low values, and changes near the bottom edge. A similar pattern is obtained from calculations of $g'_w(\xi)$ at $g_w = 0.5$ (Fig. 6). The existence of three characteristic regions in the flow near the porous surface is related to the different effects of diffusion, convection, and the external forces (pressure gradient) on flow in these regions. An increase in the thickness of the boundary layer (the distance from the zero streamline to the surface) is accompanied by a reduction in the effect of viscous forces on longitudinal momentum in the boundary-flow region. Here, the longitudinal momentum of a gas injected normal to the surface is increasingly acquired as a result of a favorable pressure gradient. Although the skin friction created by acceleration of the gas also decreases with an increase in injection rate, the rate of decay of friction due to viscosity turns out to be higher. This situation leads to the appearance of the characteristic regions in the flow. The existence of the third region is, as noted above, connected with the effect of the bottom pressure gradient.

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EFFECT OF RAREFACTION AND THE TEMPERATURE FACTOR ON THE STRUCTURE AND PARAMETERS OF SUPERSONIC UNDEREXPANDED JETS OF A MONATOMIC GAS

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The pattern of flow in an underexpanded supersonic jet discharged into a submerged space is determined in the general case by several dimensionless parameters which characterize conditions on the edge of the nozzle and in the surrounding medium. If we limit ourselves to the discharge of a monatomic gas from a sonic nozzle into the same gas, then the number of governing parameters is reduced to three — the characteristic Reynolds number $Re_L = Re_*/N^{0.5}$, the degree of expansion $N = p_0/p_\infty$, and the temperature factor $\tau = T_0/T_\infty$, where Re_* is the Reynolds number calculated from the parameters in the critical section of the nozzle, p_0 and T_0 are the stagnation temperature and pressure, and p_∞ and T_∞ are the ambient pressure and temperature [1].

At $Re_L > 10^2$, a continuous flow regime is realized. Here, the effect of viscosity and the temperature factor on flow in the initial section of the jet is restricted to the outer mixing zone. An inviscid core with a shock-wave structure (SWS), including suspended and central shocks, is preserved inside the jet. Meanwhile, the suspended shock and the mixing layer are separated from each other by a zone of inviscid flow. Values of $Re_L < 10^2$ correspond to flow regimes characterized by merging of the shock zones, the compressed layers, and the mixing layers. With a decrease in Re_L, the effect of viscosity and the temperature factor increases and propagates upflow [1-4].

The study [2] examined the effect of the temperature factor on the flow pattern and structure of an underexpanded argon jet discharged from a sonic nozzle into a submerged space. Here, $Re_L = 10^3$-3, $\tau = 1$-18, and $N = 370$-28,500. As the initial data, the authors used the density field obtained from electron—x-ray method. In the analysis of the experiments, most attention was paid to the range $Re_L = 10^3$-30.

In the present investigation, the ranges studied are expanded in the direction of smaller Re_L and large τ ($Re_L = 0.5$-10^2, $\tau = 1$-38). Based on analysis of results obtained for underexpanded argon jets at $N > 10^2$ in these ranges of Re_L and τ, we discerned two characteristic subregions in the transient flow regime, with eroded and completely degenerate SWS's. We also observed a region of flow regimes in which density decreases monotonically. This region can be regarded as intermediate between the transient regime and the free-molecular regime. We studied the effect of Re_L and τ on the structure and characteristic geometric

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